Assessing engineering student's modeling skills

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Introduction

To learn mathematics, including both computational skills and the ability to see concepts as abstract mathematical objects, is for many students a complicated process. Engineering students also learn mathematics in order to use it when studying many other subjects. The ability to use mathematics smoothly when perceiving and analyzing a situation is something closely connected to engineers' daily professions. Many duties of a professional engineer include the ability to perform some kind of mathematical modeling. Mathematical modeling is *not* a body of mathematical knowledge in the same way that Calculus or Differential Equations are, but rather a small collection of general principles which experience has proved to be helpful in the process of applying mathematical know-how to analyze problems that arise in various non-mathematical disciplines.

The CDIO clearly states the importance of modeling at several places in the curriculum. In the section 2: Personal and professional skills and attributes, we find the following under 2.1.2:

Modeling	Assumptions to simplify complex systems and environment
	Conceptual and qualitative models
	Quantitative models and simulations

Even so, curriculum document's statements about the teaching of mathematical modeling and applications in general describe intentions. Without valid student assessment practices, the actual achievements are never compared with the intentions and evaluated in a legitimate way. A group of researchers in mathematics education who are spread around the world in Australia, England, and Ireland, and concerned about how to detect and recognize students modeling achievement, have devised assessment strategies and a mathematical-modeling test for measuring general and specific competencies in modeling and applications (see Appendix for the test).

The mathematical-modeling test used in this study is intended to collect evidence of growth in mathematical modeling competencies (Izard, Haines, Crouch, Houston & Neill, 2003). Reports of research about these assessment approaches and procedures have been presented at ICTMA conferences since 1991. For more information about the ICTMA community, and how to obtain a huge variety of examples of applied mathematics, please see http://www.infj.ulst.ac.uk/ictma/.

Mathematical modeling

Model and *modeling* are common expressions with many seemingly different meanings. We are introduced to new car models that we are supposed to feel attracted to, to picture ourselves in possession of the new car. Architects use models of a landscape or a house to illustrate a product they want to sell. In the fashion industry, a model is a person who wears clothes that other people watching can imagine themselves wearing. Children use many models of reality in their toy cars, dolls, trains, and so forth. All modeling activities have at least two aspects in common: They use a model in order to think about or introduce the related reality, and the model is something more or less idealized or simplified.

The process of *mathematical modeling* also has a variety of definitions. As used in secondary mathematics, it ordinarily entails taking a situation, usually one from the real world, and using variables and one or more elementary functions that fit the phenomena under consideration to arrive at a conclusion that can then be interpreted in light of the original situation. Pollak (1970) argued that we seldom challenge students to study a situation and try to make a model of it for analyzing the situation.

> A carefully organized course in mathematics is sometimes too much like a hiking trip in the mountains that never leaves the well-worn trails. The tour manages to visit a steady sequence of the "high spots" of the natural scenery. It carefully avoids all false starts, dead-ends, and impossible barriers, and arrives by five o'clock every afternoon at a well-stocked cabin. The order of difficulty is carefully controlled, and it is obviously a most pleasant way to proceed. However, the hiker misses the excitement of risking an enforced camping out, of helping locate a trail, and of making his way cross-country with only intuition and a compass as a guide. "Cross-country" mathematics is a necessary ingredient of a good education. (p. 329)

Prospective engineers need to understand a great variety of topics and approaches in mathematics. Today these topics include concepts, principles, methods, and procedures inside mathematics and inside many other engineering subjects. Applied mathematics as a field and the process of mathematical modeling in particular are parts of the mathematical curriculum for prospective engineers that may be broadened, enhanced and even more important in the future because of the continuous technological revolution.

As often portrayed, the first step in the mathematical modeling process is the formulation of a real-world problem in mathematical terms—that is, the construction of a *mathematical model* consisting of variables that describe the situation and equations that relate these variables. The real-world problem is then translated into a mathematical problem that is analyzed and perhaps solved. Finally, the mathematical results obtained are interpreted in the context of the original real-world situation in an attempt to answer the question originally posed (Pollak, 1970; Mason, 1988).

In Figure 1, the left-hand column represents the real world, the right-hand column represents the mathematical world, and the middle column represents the connection between the two. In the middle column, the problem is simplified and formalized, and then the mathematical results obtained are translated back into terms meaningful in the original real-world situation. In a straightforward modeling process, one might be able to go through Stages 1 through 7 in sequence. But mathematical modeling is not always straightforward, especially when realistic results are expected. There often is a tradeoff between a model sufficiently simple that a mathematical solution is feasible and one sufficiently complex that it faithfully mirrors the real-world situation. If the model originally defined is too simple to be realistic, the mathematical results may not translate into valid real-world results.

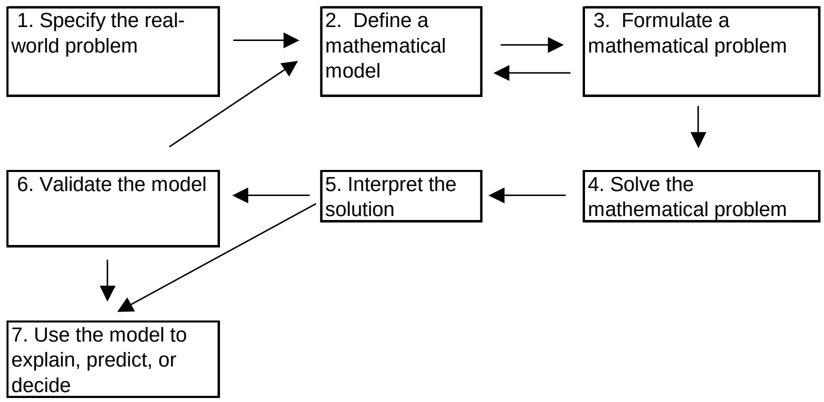

Figure 1. Main stages in modeling (adapted from Mason, 1988, p. 209).

In that case, one might have to return from Stage 6 to Stage 2 and repeat the process using a more sophisticated model. In many cases, particularly in the social sciences, it is difficult to carry out the Stage 6 validation step at all, and one might simply proceed directly from Stage 5 to Stage 7. In other cases, when the mathematical model is so sophisticated that the mathematics is intractable, one might have to return to Stage 2 and simplify the model in order to make a mathematical solution feasible. But then the validation step of Stage 6 might indicate that the model is now too simple to yield correct real-world results. There is an inevitable tradeoff, therefore, between what is physically realistic and what is mathematically possible. The construction of a model that adequately bridges this gap between realism and feasibility is the most crucial and delicate step in the process. To assess this complex process in reality is of course very difficult.

> We are prepared to risk our skin by claiming that assessment of applications and modelling is easy. As mentioned earlier, assessment is not easy if we (have to) stick to conventional modes and practices. In that case sound assessment is rather very difficult if not impossible. (Niss, 1993, p. 48)

Assessment validity

The concept of test validity for an assessment instrument is only an objective measure and evidence that the test actually measures what it purports to measure. In order to call student assessments valid, we should ask for a comprehensive list of fulfillment. The assessment procedures should provide quality assurance for certification of achievement or professional recognition, for informing management, and for evaluation of innovations and developmental interventions. Students' assessment is a powerful influence on what is taught and what will be taught and reshapes our notion of what is important in the educational program. It also gives students a better understanding of what is expected of them in a program, and in their future professions. Finally, it gives program coordinators and people at superior positions, a good hint about what students actually know (in a more general sense) after going through their programs.

The concept of validity refers, in this context, to the extent to which meaningful, appropriate, and useful inference can be made about students' responses on a test used for particular purposes. Evidence of validity for a purpose can be construct-related, content-related, and criteria-related (both concurrent and predictive evidence). The main validity issue is the extent to which meaningful, appropriate, and useful inference can be made about assessment scores.

The content in this study was curriculum related, and the problems given in the test were of types that might have been given by different teachers during the program, although the high frequency of the evaluating situations probably would be rather unusual. Thus, although this study lacks but comes close to ecological validity, it is nevertheless an experiment contrived outside normal class conditions in the Mechanical engineering program at Chalmers.

Survey1

The first survey was completed in September 2003, when the students just had started their third year of studies in the Mechanical engineering program. At this time, after at least two years in the Mechanical engineering program, the students have studied a broad variety of courses that address the topics in the mathematical-modeling test. Because of a very condensed schedule with few free openings or gaps in their schedule, the students and I met after their first lecture in a course on Electronic Control Systems in Mechanical and Electrical Engineering. The allotted test time was 45 minutes.

A total of 87 students participated and attempted to answer to the 22 questions in the Mathematical-modeling test (maximum score is 44). Fourteen were women, and 73 were men, which give the proportion of 16% women to men. The mean value for the whole group (n = 87) was 20.9 with a standard deviation equal to 6.4. As seen in Figure 2, there were two students who scored 33, and three students who scored 4, 7, and 9 respectively.

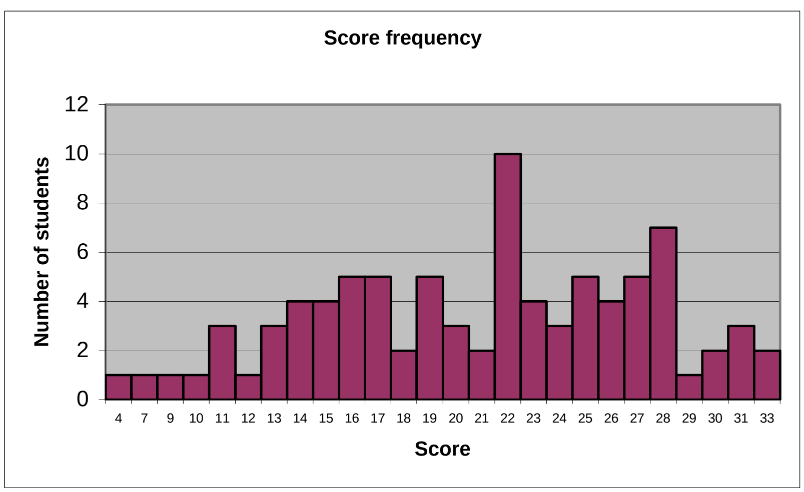

Figure 2: Score frequency for the first mathematical-modeling test in the fall of 2003.

The different results for women and men in the Mathematical-modeling test were the following:

	Women	Men
Mean value:	20.2	21.0
Standard deviation:	5.0	6.6

The most difficult questions for the 87 respondents were question number 5 (solving frequency = 29%), closely followed by question number 17. The question the students got the best score on were question number 20, with a solving frequency of 83 %. The estimated time for the test was 45 minutes and some students stayed for almost an hour, but I consider it realistic to believe that at least one third of the students needed longer time in order to do full justice to their applied mathematical capacity.

Survey2

The second survey was completed in February 2004, with the students just facing the beginning of the end of their third year. At the time of the test, they were taking a course in Project Management. Due to the critics concerning the shortage of time at the first survey, the allotted time was 1 hour and 15 minutes.

A total of 39 students participated this time. Nine were women, and 30 were men, which give the proportion of 23% women to men. The mean value for the whole group (n = 39) was 25.8 with a standard deviation equal to 5.1. As seen in Figure 3, there were six students who scored 31, and three students who scored 11, 12, and 12 respectively.

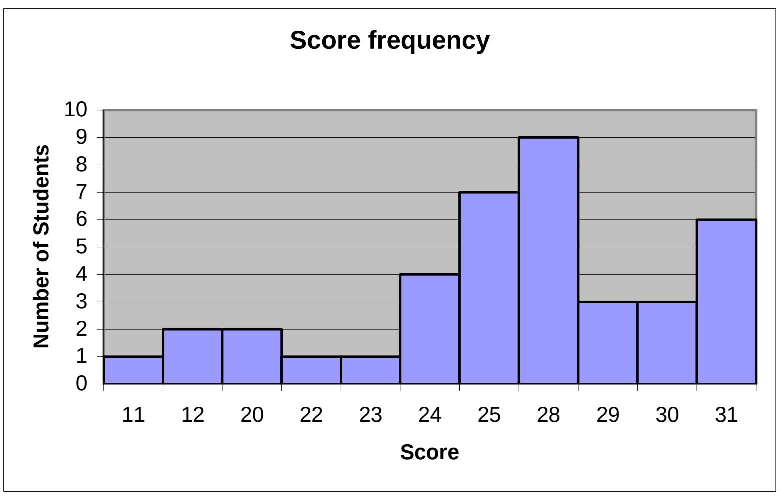

Figure 3: Score frequency for the second mathematical-modeling test in the spring of 2004.

The different results for women and men in the second Mathematical-modeling test were the following:

	Women	Men
Mean value:	20.8	27.3
Standard deviation:	8.0	2.6

The figures are in some sense devastating, while the men's results has gone up significantly, the girls results are about the same, although the distribution is even larger than in the first study. It is also a result from the fact that some of the women scored very well (three of them scored 30), but altogether it is surprising that the difference between women and men had increased between survey 1 and survey 2.

In order to obtain some external validity, I gave the same test to prospective teachers who study mathematics and natural science to become gymnasium teachers (for Grade 10-12). A total of 20 students participated. Nine were women, and 11 were men. The mean value for the whole group (n = 20) was 23.5 with a standard deviation equal to 5.7. There were three students who scored 30, one who scored 32, and three students who scored 12, 14, and 17 respectively. There were no significant differences between women and men in that group, which in a way makes the results from the two Chalmers surveys even more interesting.

The mathematical-modeling test

Multiple-choice tests are normally used for large samples of students because they are easy and quick for the students to respond to and often easy to grade. Every multiple-choice test is built around one introductory question or incomplete statement at the beginning of each item, which is followed by several options. The *options* consist of the answer -- the correct option – and several *distracters* - the incorrect but in general equally tempting options. A multiple-choice test needs to balance the correct and the incorrect answers in a way that the respondents hopefully experience as equal likely.

To write a multiple choice test that takes into consideration all different aspects of mathematical modeling is of course a truly hard task. The test used in this survey consists of eight different phases in the process of mathematical modeling; each defined below and exemplified by a specific question in the test.

Mathematical-modeling phase	**Question**
Making simplifying assumptions	2
Clarifying the goal	5
Formulating the problem	8
Assigning variables, parameters, and constants	11
Formulating mathematical statements	14
Selecting a model	18
Using graphical representation	20
Relating back to the real situation	21

Every one of the 22 different problems has one correct answer, worth 2 score points, and at least one “almost” or “nearly” correct answer worth 1 point. It means that the maximum score when students have all the problems correct is 44 score points.

The Mathematical validity of the test

When using a multiple-choice test with 5 different possibly answers, one must check what chance there is to actually guess a given amount of correct answers. This falls under the binomial probability theorem. It is easy to conclude that the probability to guess 10 correct answers is about 0.00455. Obviously that we do not need to worry much about the probability that someone will score well on the survey by simply guessing for instance the correct answers of 10 questions.

References

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Appendix **Mathematical Modelling test**

Think of yourself as a mathematical modelling person and try to answer the following questions. It is a multiple-choice test and will take you a few minutes to complete. Please mark the correct answer of the five options A, B, C, D and E on the separate answer sheet.

1. Consider the real world problem (do **not** try to solve it!):
A bus stop position has to be placed along a road on a new bus route. A covered shelter will be provided. Where should the stop be placed so that the greatest number of people will be encouraged to use the service? The bus company wants people to use the service but cannot lay on buses on demand.

Which **one** of the following assumptions do you consider the **least** important in formulating a simple mathematical model?

A. Assume that just one bus shelter will be erected
B. Assume that the road is straight
C. Assume that the weather is twice as likely to be dry as it is to be wet
D. Assume that the bus runs to a half-hourly timetable
E. Assume that customers will not walk great distances to catch a bus

2. Consider the real world problem (do **not** try to solve it!):
A tram stop position has to be placed along a new tram route. A covered shelter will be provided. Where should the stop be placed so that the greatest number of people will be encouraged to use the service? The transport company wants people to use the service but of course cannot lay on trams on demand.

Which **one** of the following assumptions do you consider the **least** important in formulating a simple mathematical model?

A. Assume that tram passengers will not walk great distances to catch a tram
B. Assume that the trams run to a twenty-minute timetable
C. Assume that the tramline is single track
D. Assume that the tram driver can drive the tram from either end of the tram
E. Assume that the tram stop *could* be placed at any position

3. Consider the real world problem (do **not** try to solve it!):
A pedestrian crossing is being considered for a busy road. Assume that the road is a straight one-way single carriageway.

Which **one** of the following assumptions do you consider the **least** important in formulating a simple mathematical model which would determine whether the crossing was needed?

A. The crossing will be controlled by buttons pushed by users
B. The density of traffic is constant
C. The speed of traffic is constant and equal to the speed limit
D. Pedestrians cross at a constant rate
E. Pedestrians will not walk long distances to use it

This mathematical modeling test was developed and improved throughout a joint collaboration effort by Ros Crouch at University of Hertfordshire, U. K., John Davis at , A. Fitzharris, at , Chris Haines at City University, U. K., John Izard, RMIT University, Australia, and, Ken Houston, & Neville Neill, University of Ulster, N Ireland.

4. Consider the real world problem (do **not** try to solve it!):

What is the best size for bicycle wheels?

Which **one** of the following clarifying questions most addresses the smoothness of the ride?

A. Are the wheels connected to the pedals by a chain?
B. How tall is the rider?
C. Has the bicycle got gears?
D. How high is the highest kerb that can be ridden up?
E. Does terrain matter?

5. Consider the real world problem (do **not** try to solve it!):

What is the best size for pushchair wheels?

Which **one** of the following clarifying questions most addresses the smoothness of the ride as felt by the child?

A. Does the pushchair have three or four wheels?
B. What is the distance between the front and the back wheels?
C. Is the seat padded?
D. How old is the child?
E. Is the pavement tarmac or paving slabs?

6. Consider the real world problem (do **not** try to solve it!):

You wish to reverse your car into a gap in a line of parked cars. The space available is approximately half as long again as the length of your car. Which ***one*** *of the following variables is most important in carrying out the manoeuvre successfully?*

A. The turning radius of the car
B. The distance you drive past the space
C. The prevailing weather conditions
D. Whether or not you can mount the kerb
E. The distance between your car and the parallel parked cars when beginning to reverse

7. Consider the real world problem (do **not** try to solve it!):

An airline requires travellers to check-in for flights at a large number of check-in positions at an airport. This, at busy times, leads to frustratingly long waiting times for passengers. Should the airline introduce a single queue for its travellers with the first person in the queue being called to a vacant check-in as it arises or should travellers form queues at each check-in position?

In the following unfinished problem statement which **one** of the five options should be used to complete the statement?
Please ring the correct answer of the five options A, B, C, D and E.

Given that there are ten check-in positions and **given** that travellers arrive at the check-ins at regular intervals with a random amount of baggage, **find** by simulation methods the average waiting time for each traveller where queues are formed at each check-in and **compare** it with the

A. average waiting time for each traveller where queues are formed at 5 check-ins and where a single queue is used for the remaining five check-ins.

B. average waiting time for each traveller where a single queue is used for the 10 check-ins.

C. average waiting time for each traveller where queues are formed at some check-ins and where a single queue is used for the remaining check-ins.

D. average waiting time for each traveller where queues are formed at 8 check-ins and where a single queue is used for the remaining 2 check-ins.

E. average waiting time for each traveller where queues are formed at 2 check-ins and where a single queue is used for the remaining 8 check-ins.

8. Consider the real world problem (do **not** try to solve it!):

A large supermarket has a great many sales checkouts, which at busy times lead to frustratingly long delays especially for customers with few items. Should express checkouts be introduced for customers who have purchased fewer than a certain number of items?

In the following unfinished problem statement which **one** of the five options should be used to complete the statement?

Given that there are five checkouts and **given** that customers arrive at the checkouts at regular intervals with a random number of items (less than 30), **find** by simulation methods the average waiting time for each customer at 5 checkouts operating normally and **compare** it with

A. the average waiting time for each customer at 1 checkout operating normally whilst the other 4 checkouts are reserved for customers with 8 items or less.

B. the average waiting time for each customer at 4 checkouts operating normally whilst the other checkout is reserved for customers with fewer items.

C. the average waiting time for each customer at 1 checkout operating normally whilst the other 4 checkouts are reserved for customers with fewer items.

D. the average waiting time for each customer at some checkouts operating normally whilst other checkouts are reserved for customers with 8 items or less.

E. the average waiting time for each customer at 4 checkouts operating normally whilst the other checkout is reserved for customers with 8 items or less.

9. Consider the real world problem (do **not** try to solve it!):

A high street bank has a number of teller-windows at which business may be conducted. Some customers have only one transaction to complete, for example, cashing or lodging a cheque. Other customers have several items of business to transact which may take a long time, such as lodging many bags of coins. Should the bank have a single queue system for serving customers or should it reserve some windows for customers with just a small number of transactions?

In the following unfinished problem statement which **one** of the five options should be used to complete the statement?

Given that there are six teller windows and **given** that customers arrive in the bank at regular intervals with a random number of transactions to complete, **find** by simulation methods the average waiting time for each customer when there is a "single queue **-** six server" system and compare it

A. the average waiting time for each customer where separate queues are formed at each window

B. the average waiting time for each customer where separate queues are formed at some windows and a single queue is used for the remaining windows

C. the average waiting time for each customer where a "fast line" queue is formed at one window and a single queue is used for the remaining five windows

D. the average waiting time for each customer where a "fast line" queue is served by some windows and a “slow line" queue is served by the other windows

E. the average waiting time for each customer where queues are formed at two windows and a single queue uses the remaining windows

10. Consider the real world problem (do **not** try to solve it!):

The time required to evacuate an office block in an emergency needs to be known by the safety officer. There are conflicting needs of security, public access and ease of exit.

In a simple mathematical model, a single room is considered with people exiting that room in single file. Which **one** of the following options contains parameters, variables or constants, each of which should be included in the model?

A. Time elapsed after the alarm raised: Number of people evacuated at time t: Time of day at which the alarm sounded

B. Number of people to be evacuated: Time elapsed after the alarm raised: Number of people evacuated at time t

C. Number of people evacuated at time t: Time of day at which the emergency occurred: Width of the emergency exits

D. Total time to evacuate everyone: Space between people leaving: Width of the emergency exits

E. Speed of the line of people: Initial delay before the first person can leave: Amount of personal belongings carried out

11. Consider the real world problem (do **not** try to solve it!):

The time required to evacuate an aircraft after an emergency landing at an airport needs to be known by the emergency and safety services. There are conflicting needs of aircraft construction, safety, access and ease of exit.

In a simple mathematical model, an aircraft fuselage wide enough for two seats either side of a central aisle is considered with passengers exiting singly at the front and the rear of the aircraft. Which **one** of the following options contains parameters, variables or constants, each of which should be included in the model?

A. Time elapsed after the emergency landing: Number of people evacuated at time t: Time of day at which the landing occurred

B. Speed of people leaving their seats: Initial delay in unbuckling seatbelts before the first person can leave: Amount of personal items carried out

C. Number of people evacuated at time t: Time of day at which the landing occurred: Width of the emergency exits

D. Total time to evacuate everyone: Space between passengers leaving: Width of the emergency exits

E. Number of people in the aircraft: Time elapsed after the emergency landing: Number of people evacuated at time t

12. Consider the real world problem (do **not** try to solve it!):

The University holds regular fire drills to estimate emergency evacuation times. Consider the situation in which students leave a laboratory in single file.

Which **one** of the following options contains parameters, variables or constants each of which should be included in a mathematical model of the evacuation?

A. Time elapsed after the alarm sounded: Number of students evacuated at time t: Whether the evacuation was in the morning or afternoon.

B. Total number of students to be evacuated: Time elapsed after the alarm sounded: Number of students evacuated at time t.

C. Number of students evacuated at time t:: Whether the evacuation was in the morning or afternoon: Width of the laboratory doors.

D. Total time to evacuate all students: Distance between consecutive students leaving: Width of the laboratory doors.

E. Rate at which students leave the laboratory: Initial delay before the first person can leave: Quantity of bags and books carried out.

13. Consider the problem:

A ferry has a total deck space of area A. It carries cars, each car taking up an amount C of deck space, and lorries, each lorry needing an amount of L deck space. Each car pays £p for the crossing and each lorry pays £q. The manager wants to know how many cars (x) and how many lorries (y) to take on board so as to obtain the maximum revenue.

Which **one** of these options gives the revenue subject to the restriction on deck space?

A. $xp + yq$ subject to $yC + xL \leq A$

B. $xp + yq$ subject to $xC + yL \leq A$

C. $(x + y)(p + q)$ subject to $xC + yL \leq A$

D. $xp + yq$ subject to $xC + yL = A$

E. $(x + y)(p + q)$ subject to $(x + y)(C + L) \leq A$

14. Consider the problem:

There are two queues at a supermarket checkout. In the first queue there are m_1 customers all with n_1 items in their baskets, while in the second queue there are m_2 customers all with n_2 items in their baskets. It takes t seconds to process each item and p seconds for each person to pay and customers wish to know which queue to join.

Which **one** of these options gives the condition for the first queue to be the better queue to join?

A. $m_1(p+n_1t) = m_2(p+n_2t)$

B. $m_1(p+n_1t) < m_2(p+n_2t)$

C. $m_2(p+n_2t) \leq m_1(p+n_1t)$

D. $m_2(p+n_2t) < m_1(p+n_1t)$

E. $m_1(p+n_1t) \leq m_2(p+n_2t)$

15. Consider the problem

New printers are to be purchased for the Computer Services terminal room. The Alpha printer costs £p each and the Beta printer £q each. The Alpha needs r m^2 of floor space and the Beta s m^2. Total floor space available is t m^2. At least b of each type must be bought and the total budget must not exceed £A.

Which **one** of these options models the situation mathematically if x is the number of Alpha printers and y the number of Beta printers?

A. $x \geq b,\ y \geq b$
$xr + sy \leq t$
subject to $px + qy \leq A$

B. $x > b,\ y > b$
$xr + sy < t$
subject to $px + qy \leq A$

C. $x \geq b,\ y \geq b$
$xs + ry \leq t$
subject to $py + qx \leq A$

D. $x > b,\ y > b$
$(x+y)(r+s) \leq t$
subject to $(p+q)(r+s) = A$

E. $x \leq b,\ y \leq b$
$xr + sy \geq t$
subject to $px + qy \geq A$

16. Which **one** of the following options most closely models the height of a sunflower while it is growing (in terms of time t)?

A. $1 - e^{-t}$ B. $(1 - t)^2$ C. t

D. $t - t^2$ E. $\frac{1}{1 + e^{-t}}$

17. Which **one** of the following options most closely models the speed of a car starting from rest (in terms of time t)?

A. $1 - e^{-t}$ B. $(1 - t)^2$ C. t

D. $t - t^2$ E. $\frac{1}{1 + e^{-t}}$

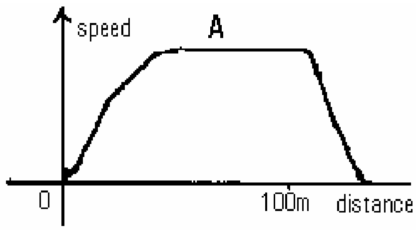
18. Which **one** of the following options most closely models the distance fallen by an object released from a tall building (in terms of time t)?

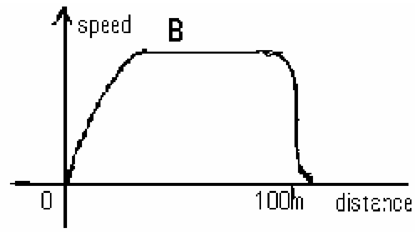
A. $e^{5t} - 1$ B. $(1 - 5t)^2$ C. $5t$ D. $5t^2$ E. $\frac{1}{1 + e^{5t}}$

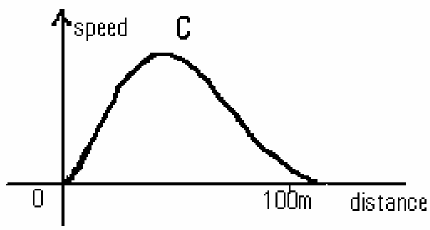
19. The following situation has been partially modelled:

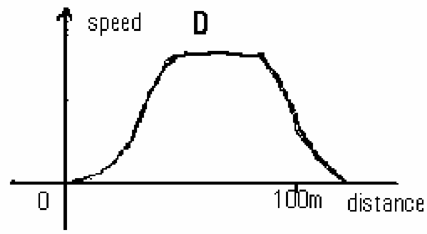
An Olympic 100m sprinter wins his race, waves to the crowd and stops to watch the video replay on the giant screen in the stadium.

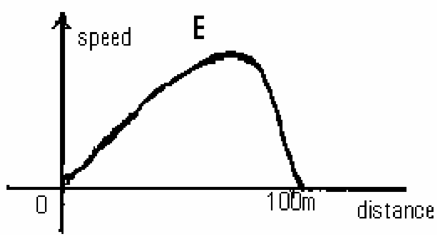
Which **one** of the following graphs best represents the variation in the speed of the runner over the distance covered from the start of the race to the end of the video replay?







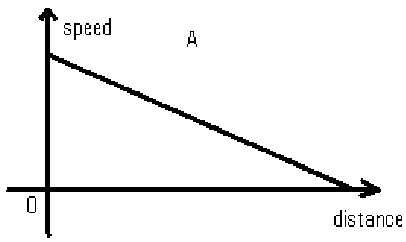


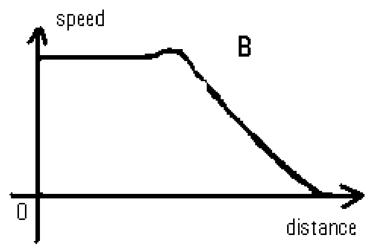


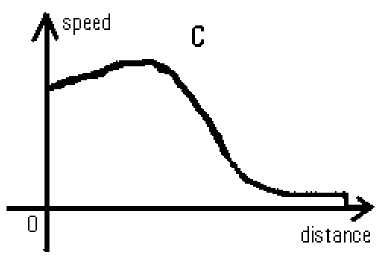
20. The following situation has been partially modelled:

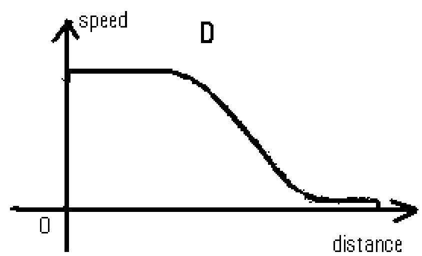
An aircraft is waiting to land at a busy airport. It has been stacked at a constant height flying on an approximately circular path at a fixed speed. At a particular moment the aircraft is instructed to land and to taxi some distance to the airport terminal.

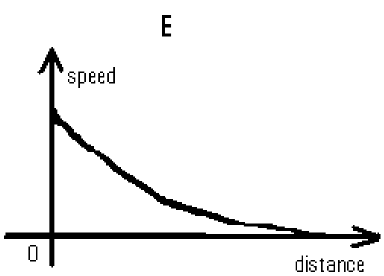
Which **one** of the following graphs best represents the variation in the speed of the aircraft as the distance covered increases, from the stacking situation to the arrival at the terminal?







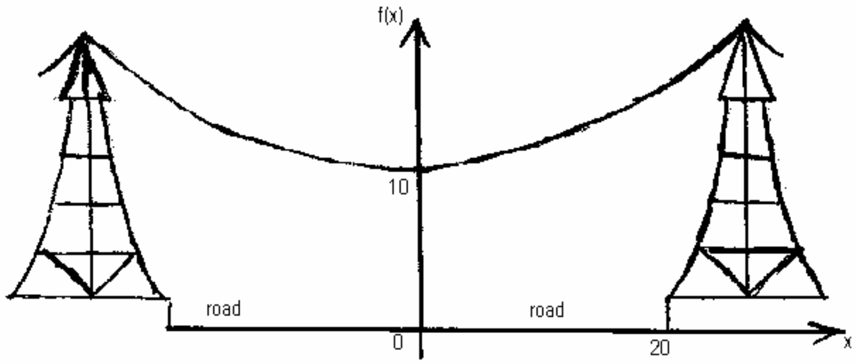




21. The following situation has been partially modelled:

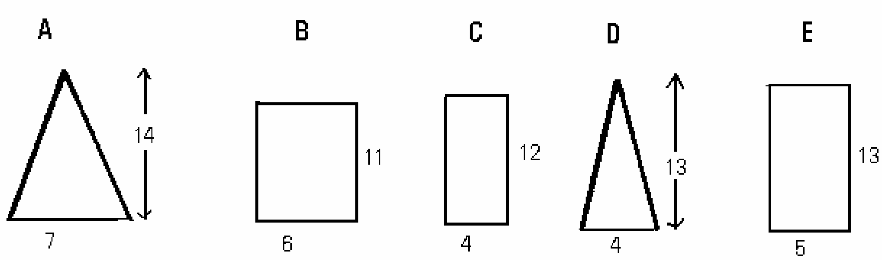
An electricity high voltage cable is supported by two pylons on either side of a busy two-way road, which places restrictions on the height and size of objects/vehicles that can pass safely beneath it.

In this diagram, which is symmetric, the carriageway is shown to be 20m wide in each direction and the cable is modelled by the function 10f(x), which takes the values (metres) given in the table below:



x	12	13	14	15	16	17	18	19	20
F(x)	1.152	1.180	1.209	1.242	1.276	1.314	1.354	1.397	1.442

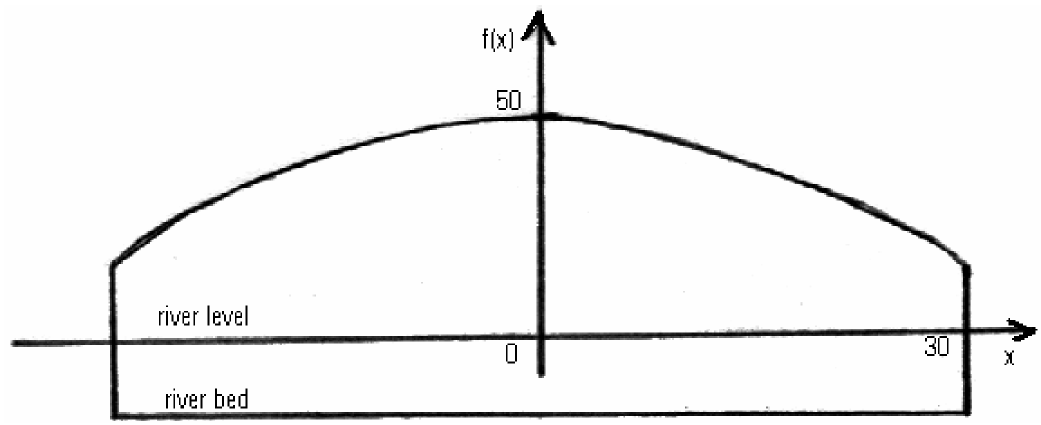
Which **one** of the following objects can pass beneath the cable on a low loader, assuming that the base of the object is one metre above 'road level'?



22. The following situation has been partially modelled:

A bridge across a wide river places restrictions on the height and size of objects/structures that can safely pass beneath it on a river barge.

In this diagram, which is symmetric, the river is shown to be 60m wide and the underside of the bridge is modelled by a function f(x), which takes the values (metres) given in the table:



x	4	5	6	7	8	9	10	11	12
f(x)	49.29	48.89	48.40	47.82	47.16	46.40	45.56	44.62	43.60

Which **one** of the following objects can pass beneath the bridge on a barge, assuming that the base of the object is 2 metres below the level of the river?